

Oral Exams in Geometry and Topology

All-round (Solve 1 out of 2 problems)

1. A 2-form ω on a manifold X is said to be non-degenerate if the contraction with ω is an isomorphism $TM \cong T^*M$. Then a smooth function H on X produces a vector field V_H , which corresponds to dH under the above isomorphism $TM \cong T^*M$.

- (1) For example, show that the standard area form on \mathbb{R}^2 is non-degenerate. As an example, let $H = x^2 + y^2$. Compute the vector field V_H . What is the flow generated by this vector field?
- (2) Note that the directional derivative $V_H \cdot H$ is zero in the above example. Prove that this is generally true.
- (3) Note that the area form ω is invariant under the flow in the above example. Prove that this is generally true, if we further assume the non-degenerate two-form ω is closed.

2. Show that any compact odd-dimensional Riemannian manifold with positive sectional curvature is orientable.